Censored Quantile Regression

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Abstract

Quantile regression methods offer a powerful and natural approach to analyzing statistical variability caused by heteroscedasticity in the data or inhomogeneity in the population. This is especially true for survival analysis for two reasons. First, the quantiles of the survival function are often of fundamental importance and so direct estimation of the quantiles is especially natural. Second, standard models (for example, the proportional hazards model) require that any effect in the conditional quantiles be a monotonic function of the quantile function corresponding to the baseline hazard. This precludes behavior where the sign of the effect may change with the size of the response. For example, a proportional hazards model does NOT permit a treatment effect to be significantly positive for more severe cases (with short survival times) and not positive in all other cases. To detect such reversals, it may be possible to find additional time-varying covariates, or to attempt other modelling approaches; but conditional quantile methods offer a rather natural approach to this problem.

Until 2004, this approach had been hampered by not having a conditional quantile estimator for censored data that is directly analogous to the Kaplan-Meier estimator for a single sample. However, by considering the Kaplan-Meier estimator as a recursively reweighted estimator of a survival function, it is possible to provide a direct generalization to the quantile regression case. In fact, the basic reweighting scheme provides a quantile estimator whose subgradient remains linear in the probability argument, and thus permits very fast computation using traditional simplex pivoting methods. More recent work has extended this approach to allow partly linear models and to provide globally robust (high-breakdown) estimators. A number of examples suggest the strong potential of these regression quantile methods for censored survival data.